Name: ____________________________

Math 10560, Final Exam:
May 8, 2006

Instructor: ____________________________

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

Name: ____________________________

Instructor: _________________________

Multiple Choice

1.(6 pts.) The function $f(x) = 2x + \ln x$ is one-to-one. Compute $(f^{-1})'(2)$.

(a) $\frac{1}{3}$ (b) $\frac{5}{2}$ (c) $4 + \ln 2$

(d) $\frac{2}{5}$ (e) 0

2.(6 pts.) Solve the equation $\log_4(x) + \log_4(x^2) = -\frac{3}{2}$. Then $x =$

(a) $\frac{1}{\sqrt{e}}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) (e) -2

Name: ____________________________

Instructor: ________________________

3.(6 pts.) Use logarithmic differentiation to compute the derivative of the function

$$f(x) = \frac{2^x(x^3+1)}{\sqrt{x+1}}.$$

(a) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\ln 2 + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

(b) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{2} + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

(c) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{\ln 2} + \dfrac{1}{x^3+1} - \dfrac{1}{x+1}\right)$

(d) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(2 + \dfrac{1}{x^3+1} - \dfrac{1}{x+1}\right)$

(e) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{\ln 2} + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

4.(6 pts.) You begin an experiment at 9am with a sample of 1000 bacteria. An hour later your population has doubled. Assuming exponential growth, what is the population at noon?

(a) $32,000$ (b) $4,000$ (c) $8,000$ (d) $1,000e^{-3}$ (e) $1,000e^3$

Name: ____________________________

Instructor: _________________________

5.(6 pts.) Simplify $\sin^{-1}(\sin \frac{9\pi}{10})$.

(a) $-\frac{\pi}{10}$

(b) $\frac{9\pi}{10}$

(c) not enough information to tell.

(d) $\frac{\pi}{10}$

(e) 0

6.(6 pts.) Compute the limit $\lim_{x\to\infty}(2x)^{\frac{1}{x}}$.

(a) 1 (b) 0 (c) e (d) 2 (e) ∞

Name: ____________________________

Instructor: ________________________

7.(6 pts.) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} x\sin(x)dx.$$

(a) 0 (b) 1 (c) -1 (d) π (e) $\frac{\pi}{2}$

8.(6 pts.) Find the integral $\int_0^2 \sqrt{4-x^2}dx$.

(a) 0 (b) $4+\sin 4$ (c) π (d) $\pi+2$ (e) -2

Name: ______________________________

Instructor: __________________________

9.(6 pts.) Evaluate the integral

$$\int \frac{x-9}{x^2+3x-10}\,dx.$$

(a) $\ln\left|\frac{(x-5)^2}{x+2}\right|+C$ (b) $\ln\left|\frac{x-2}{x+5}\right|+C$ (c) $\ln\left|\frac{(x+2)^2}{(x-5)^3}\right|+C$

(d) $\ln\left|\frac{(x+5)^2}{x-2}\right|+C$ (e) $\ln\left|\frac{x+5}{(x-2)^2}\right|+C$

10.(6 pts.) Determine whether the following integral converges or diverges. If it converges, evaluate.

$$\int_{-2}^{0} \frac{1}{(x+1)^2}\,dx.$$

(a) Converges to -2. (b) Converges to 0. (c) Converges to 2.

(d) Diverges. (e) Converges to 1.

Name: ______________________________

Instructor: __________________________

11.(6 pts.) Find the length of the curve

$$y = 1 + \frac{4}{3}x^{3/2} \qquad \text{for } 0 \leq x \leq 1.$$

(a) $\frac{1}{6}(1-\sqrt{5})$ (b) $\frac{2}{3}(1-\sqrt{5})$ (c) $\frac{1}{12}(3\sqrt{3}-1)$

(d) $\frac{1}{12}(11\sqrt{5}-1)$ (e) $\frac{1}{6}(5\sqrt{5}-1)$

12.(6 pts.) Find the centroid of the region bounded by $y = x^2$ and $y = x$.

(a) $(\frac{1}{2}, \frac{1}{2})$ (b) $(\frac{1}{10}, \frac{2}{5})$ (c) $(\frac{1}{10}, \frac{1}{15})$

(d) $(\frac{1}{12}, \frac{1}{15})$ (e) $(\frac{1}{2}, \frac{2}{5})$

Name: ______________________________

Instructor: __________________________

13.(6 pts.) The solution to the initial value problem

$$y' = \frac{\sin(x)}{2y+1} \qquad\qquad y(0) = 2$$

satisfies

(a) $2y + 1 = 6 - e^{-\cos x}$ (b) $y^2 + y = 7 - \cos x$

(c) $y^2 + y = 6\cos x$ (d) $2y + 1 = 5e^{-\cos x}$

(e) $e^{2y+1} = e^5 + \arcsin x$

14.(6 pts.) The solution to the initial value problem

$$\frac{dy}{dx} + xy + x = 0 \qquad y(0) = 0$$

is

(a) $y = e^{-\frac{x^2}{2}} - 1$ (b) $y = e - e^{-\frac{x^2}{2}+1}$ (c) $y = xe^x$

(d) $y = 1 - e^{-x}$ (e) $y = e^{-x} - 1$

Name: ______________________________

Instructor: __________________________

15.(6 pts.) Investigate the convergence or divergence of the sequence

$$\lim_{n\to\infty} (-1)^n \frac{3n^2}{n^2+1}.$$

If the sequence converges, find its limit.

(a) -3 (b) $(-1)^n 3$ (c) The sequence is divergent

(d) 3 (e) 0

16.(6 pts.) Investigate convergence or divergence of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(4-\pi)^{n-1}}{\pi^n}.$$

If the series converges, calculate its sum. Note: $4 > \pi > 3$.

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) The series is divergent

(d) $\frac{1}{4}$ (e) $-\frac{1}{4}$

Name: ______________________________

Instructor: __________________________

17.(6 pts.) The series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cos\left(\frac{1}{n}\right)$$

is

(a) absolutely convergent by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(b) conditionally convergent by root test

(c) divergent by integral test

(d) divergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(e) absolutely convergent by ratio test

18.(6 pts.) Which of the following series converge conditionally?

$$(1) \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n}+1}; \qquad (2) \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}+1}; \qquad (3) \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^{5/2}+1}.$$

(a) (1) and (2) converge conditionally, (3) does not converge conditionally

(b) (2) converges conditionally, (1) and (3) do not converge conditionally

(c) (1) converges conditionally, (2) and (3) do not converge conditionally

(d) (1) and (3) converge conditionally, (2) does not converge conditionally

(e) (3) converges conditionally, (1) and (2) do not converge conditionally

Name: ______________________________

Instructor: __________________________

19.(6 pts.) The series

$$\sum_{n=1}^{\infty} \frac{8^n}{n^2}(x-1)^{3n}$$

has the radius of convergence

(a) 0 (b) $\frac{1}{2}$ (c) ∞ (d) 1 (e) $\frac{1}{8}$

20.(6 pts.) Consider the Taylor series of

$$f(x) = \sum_{n=1}^{\infty} \frac{n^n}{n!} x^n.$$

Find $f^{(100)}(0)$.

(a) $\frac{100^{100}}{((100)!)^2}$ (b) $\frac{(100)!}{100^{100}}$ (c) $\frac{100^{100}}{(100)!}$

(d) $(100)!$ (e) 100^{100}

Name: ____________________________

Instructor: __________________________

21.(6 pts.) Determine for which x the approximation of $\sin x$ by its 3rd degree MacLaurin polynomial $T_3(x)$ (Taylor polynomial centered at 0) is accurate to within $\frac{1}{3840}$, by using the Alternating Series Remainder Estimation Theorem.
Note: $3840 = 120 \cdot 2^5$.

(a) $-\sqrt{32} < x < \sqrt{32}$

(b) $-1 < x < 1$

(c) $-\sqrt[5]{120} < x < \sqrt[5]{120}$

(d) $-120 < x < 120$

(e) $-\frac{1}{2} < x < \frac{1}{2}$

22.(6 pts.) Let $x = \sin(9t)$ and $y = \cos(9t)$. Then $\frac{dy}{dx} =$

(a) $\tan(9t)$

(b) $-\tan(9t)$

(c) $9\tan(t)$

(d) $81\sec^2(9t)$

(e) $\cot(9t)$

Name: ______________________________

Instructor: __________________________

23.(6 pts.) The point $(2, \dfrac{13\pi}{6})$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(\sqrt{3}, 1)$

(b) $(-\sqrt{3}, 1)$

(c) $(1, \sqrt{3})$

(d) $(-1, \sqrt{3})$

(e) Since $\dfrac{13\pi}{6} > 2\pi$, there is no such point.

24.(6 pts.) Which integral below gives the surface area of the surface of revolution obtained by rotating the polar curve $r = \sin\theta$, $0 \le \theta \le \pi$ about the x-axis?
Hint: A polar curve is also a parameterized curve.

(a) $2\pi \displaystyle\int_0^{\pi} \sin\theta \ \cos^2\theta \, d\theta$

(b) $2\pi \displaystyle\int_0^{\pi} \cos^2\theta \, d\theta$

(c) $2\pi \displaystyle\int_0^{\pi} \sin^2\theta \, d\theta$

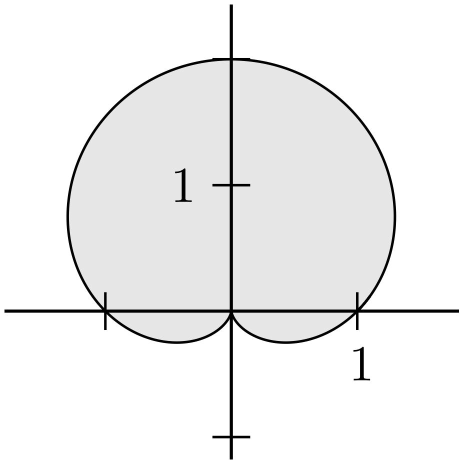
(d) $\dfrac{\pi}{2} \displaystyle\int_0^{\pi} \sin\theta \ \cos^2\theta \, d\theta$

(e) $2\pi \displaystyle\int_0^{\pi} \sin\theta \ \cos\theta \, d\theta$

Name: ______________________________

Instructor: __________________________

25.(6 pts.) Find the area inside the cardioid $r = 1 + \sin\theta$.



(a) $\frac{3}{2}$ (b) 2π (c) 2 (d) $3\pi + \ln 4$ (e) $\frac{3\pi}{2}$

Name: ____________________

Math 10560, Final Exam:
May 8, 2006

Instructor: ANSWERS

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)	16.	(a)	(b)	(c)	(•)	(e)
3.	(•)	(b)	(c)	(d)	(e)	17.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(•)	(d)	(e)	18.	(a)	(b)	(•)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)	19.	(a)	(•)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(•)
7.	(a)	(•)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(b)	(•)	(d)	(e)	22.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(•)	(e)	23.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)	24.	(a)	(b)	(•)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(•)	25.	(a)	(b)	(c)	(d)	(•)
12.	(a)	(b)	(c)	(d)	(•)						
13.	(a)	(•)	(c)	(d)	(e)						
14.	(•)	(b)	(c)	(d)	(e)						